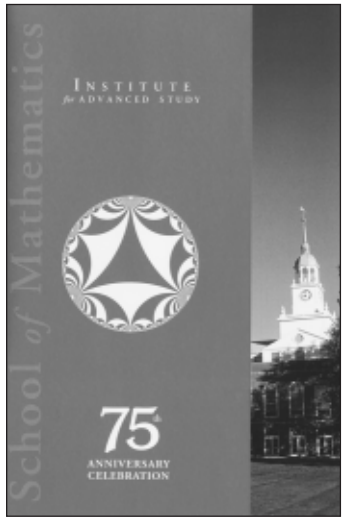
The IAS School of Mathematics at 75

Susan Friedlander and Mark Goresky



This year the Institute for Advanced Study (IAS) in Princeton, New Jersey, celebrates the seventy-fifth anniversary of its founding. The "Peerless Institute," to quote Allyn Jackson's article [1], "has had an enormous influence on the development of mathematics, especially in America." Endowed in 1930 by Louis Bamberger and Carrie Bamberger Fuld, the IAS was created as a sort of ivory tower for basic research, along the lines described by Abraham Flexner in his lectures and writings [2]. It was an idea whose time was ripe: American science and mathematics were not considered to be competitive with the cutting edge developments in Europe. Flexner's drive, the Bambergers' foresightful generosity, Oswald Veblen's scientific leadership, and the appalling level of anti-Semitism in Europe assured the rapid assemblage of a stellar mathematics faculty at the IAS. By 1934 the faculty included James Alexander, Albert Einstein, Marston Morse, Oswald Veblen, John Von Neumann, and Hermann Weyl.

Einstein, Weyl, and Veblen decided from the beginning that the IAS would have a much greater impact with the establishment of a "vigorous" visitor's program. By October 1933 the institute already had over twenty visitors, or "workers", later referred to as "members".

The outstanding success of the IAS over the next seventy-five years in terms of the achievements of its faculty and members was highlighted in lectures and discussions during the weekend of March 11–12, 2005. The event was well attended by an eclectic group, including current and former members, faculty, and friends of the IAS. A series of distinguished mathematicians emphasized the importance of the institute in fostering their careers and in the development of twentieth-century mathematics.

The March 11 lectures were delivered to an overflow audience in Wolfensohn Hall, the large lecture/performance hall on the IAS campus. Peter Sarnak (professor, Courant Institute of Mathematical Sciences and Princeton University) lectured on "Number theory, symmetry and zeta functions". Using the single theme of zeta functions, Sarnak managed to address some of the mathematical accomplishments of Srinivasa Ramanujan, Carl Ludwig Siegel, André Weil, Atle Selberg, Enrico Bombieri, Harish-Chandra, Armand Borel, Pierre Deligne, Goro Shimura, and Robert Langlands, each of whom (except Ramanujan) is/was associated with the IAS for a significant period of time.

Avi Wigderson (Herbert Maass Professor, IAS) spoke on "Kurt Gödel, John von Neumann, and the theory of computation". The talk was devoted to explaining some of the fundamental ideas developed by these two early IAS faculty members. Wigderson described how their contributions have greatly influenced (and continue to stimulate) the theory of computation. The powerpoint presentation for this lecture is available at: `http://www.math.ias.edu/75/files/avi_wigderson.ppt`.

George Dyson (research associate, Western Washington University) gave an entertaining lecture on "Veblen's Circle: Early years of mathematics at the Institute for Advanced Study". By presenting old

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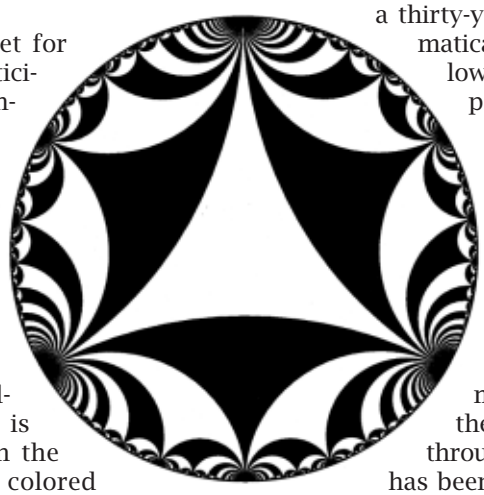
Pictured, left to right are speakers Peter Sarnak, Avi Wigderson, George Dyson, Michael Atiyah, and Raoul Bott.

photographs and letters from the IAS archives, Dyson explored Veblen's enormous influence on the founding and early development of the IAS.

On Saturday afternoon the crowd of participants once again squeezed into Wolfensohn Hall to hear lectures by Sir Michael Atiyah ("Solitons and symmetry"), Friedrich Hirzebruch ("My joint work with Armand Borel, 1952–1954), Raoul Bott ("What Morse missed by not talking to Weyl"), Weinan E ("A mathematical theory of solids—from atomic to macroscopic scales"), and Peter Ozsvath ("Heegard diagrams and holomorphic disks"), each of which was addressed to a wide audience. Videos of these lectures are expected to appear eventually on the IAS website.

On Friday evening a banquet for over one hundred (paying) participants was served in the IAS dining hall. It was prepared by the institute's talented chef, Michel Reymond, and his dedicated staff. The dessert featured an unusual chocolate cookie sporting the IAS-75 School of Mathematics logo, apparently the product of a PostScript-to-chocolate printer. A (non-chocolate) version of the logo illustrates this page. The figure is the paving of the unit disk in the complex plane by alternately colored fundamental domains of the principal congruence subgroup $\Gamma(2)$ of level two. This is one of the basic examples in the subject of modular forms which, together with closely related phenomena in representation theory and number theory, has been of central interest at the IAS since its founding.



During the morning of March 12, James Simons (Renaissance Technologies Corporation) chaired a panel discussion with Sir Michael Atiyah (University of Edinburgh), Raoul Bott (Harvard University), Fritz Hirzebruch (Max-Planck-Institut in Bonn), Weinan E (Princeton University), and Peter Ozsvath (Columbia and Berkeley). Bott recalled that his invitation to the IAS carried the expectation that he would use this time to write a book on electrical network theory. Soon after arriving (1949), Bott discovered that the book project was incompatible with his desire to pursue the many mathematical opportunities that Princeton offered. Marston Morse quickly assured him that he was absolutely free to do as he pleased. The result, Bott said, was that "my graduate education occurred here—and Jean-Pierre Serre was my tutor." Peter Goddard, director of the IAS, commented that the financial independence of the institute allows it to provide this kind of academic freedom.

Fritz Hirzebruch referred to the four academic years he spent at the IAS as the most important period of his mathematical development, as well as the venue for his marriage. On hearing that Hirzebruch would be living in Bonn, Serre was surprised and commented, "Who is in Bonn except Adenauer?" Hirzebruch took this to heart and began a thirty-year project to create a mathematical research institute in Bonn following the IAS model of an independently funded institution designed primarily to support the advanced research of postdoctoral scholars without teaching obligations. Although it differs from the School of Mathematics at the IAS in some respects, the Max-Planck-Institut für Mathematik in Bonn has been enormously successful in promoting the development of mathematics throughout Germany. In fact, the IAS has been the "mother" for many institutes worldwide, although, as Langlands observed, even the "mother" may have had a mother of her own, as Weyl's early leadership at the IAS was much influenced by Hilbert's institute at Göttingen.

Sir Michael Atiyah first came to the IAS fifty years ago and eventually spent a total of seven years at the IAS. During these periods he encountered I. M. Singer, J-P. Serre, K. Kodaira, R. Bott, and F. Hirzebruch. He wrote papers and became good friends with each of these mathematicians. He observed that collaborations established at the IAS continue through generations and diffuse around the world.

Weinan E and Peter Ozsvath, as representatives of a younger generation of mathematicians, discussed the crucial impact of the IAS on their careers. For Ozsvath it was "the institute for advanced lunch" (another testament to the excellence of




Left to right, speakers Friedrich Hirzebruch, Peter Ozsvath, Weinan E, and IAS director Peter Goddard.

IAS food!). During his two years as a member at the IAS, he began his very fruitful collaboration with Zoltan Szabo, Thomas Mwroka, and Peter Kronheimer. This illustrates that the words of I. M. Singer written in 1976 remain true to this day: "The members' stay at the IAS under the guidance of the faculty affects their mathematical careers enormously. Their contacts with their peers continue for decades. They leave the Institute, disperse to their universities, and carry with them a deeper understanding of mathematics, higher standards for research, and a sophistication hard to attain elsewhere."

References

[1] A. JACKSON, The IAS School of Mathematics, *Notices* **49** (2002), 896–904.

[2] A. FLEXNER, *Universities, American, English, German*, Oxford University Press, New York, 1930.

[3] A. BOREL, The School of Mathematics at the Institute for Advanced Study, *A Century of Mathematics in America,* vol. 3, Amer. Math. Soc., Providence, RI, 1989, pp. 119–148.

Fuld Hall, IAS campus.

Abstract for Avi Wigderson's lecture:

Kurt Gödel, John von Neumann, and the Theory of Computation

The purpose of the talk was to survey some important ideas and results of these two prominent faculty members of the School of Mathematics and explain their relation to exciting research areas (past and current) in the theory of computation.

Some of the topics discussed were the following.

• The fundamental notion of *proof*, and its fundamental relation to the notion of *computation*. Gödel's Incompleteness Theorem and its relation to limits on computation and Turing's undecidability.

• The notion of *efficient computation*. Gödel's letter to von Neumann in 1954 foreseeing the major problem of the relation between finding a proof and verifying one, namely the P vs. NP problem.

• Computation taking place in biological, physical, and other natural processes. von Neumann's model of *cellular automata*, motivated by the study of self-reproduction. Conway's "Game of Life" being a universal Turing machine.

• von Neumann's study of *fault tolerant* computers and networks. Relation to persistence of information in cellular automata and to the current study of *expander graphs*. The use of deep results in number theory (of Selberg, Deligne, and others) in the efficient construction of such expanders.

• The use of *randomness* in computation, starting with the Monte Carlo algorithms of Ulam, Metropolis, von Neumann, and others. von Neumann's questions about the source of or randomness in these procedures, leading to the current research in *pseudorandom generators* and of *randomness extractors* for weak random sources.

• von Neumann's minimax theorem and his book with Morgenstern, starting off game theory. Applications of the minimax theorem in the theory of computation and current research in algorithmic game theory, joining economists and computer scientists.

Abstract of George Dyson's talk:

Veblen's Circle: Early Years of Mathematics at the Institute for Advanced Study

"What could be wiser than to give people who can think the leisure in which to do it?" economist Walter W. Stewart advised the Institute for Advanced Study's founding director, Abraham Flexner, in 1939, the year that Fuld Hall was built. The beginnings of the IAS are usually credited to Flexner's association with Louis Bamberger and his sister, Caroline Fuld, who incorporated the new institution on May 20, 1930, with an initial US$5 million endowment and a commitment to start out with a school of math.

This talk, based on unpublished material in the IAS archives, followed the development of IAS from the perspective of Oswald Veblen (1880–1960), who first arrived in Princeton in 1905 and "conceived the whole project" in the words of P. A M. Dirac. "The way to make another step forward," Veblen had written to the Rockefeller Foundation's Simon Flexner on February 23 of 1924, "is to found and endow a Mathematical Institute." Simon Flexner replied to Veblen, "I wish that sometime you might speak with my brother, Mr. Abraham Flexner, of the General Education Board." For the next thirty-six years Veblen was directly involved with the establishment and day-to-day operation of the IAS.

Dyson covered the parallels between Veblen's career and the development of the IAS, mentioning in particular Veblen's custodianship of mathematics at Princeton University, Veblen's work during WWI and WWII at the Aberdeen Proving Ground, his leadership of the Emergency Committee for Displaced German Scholars, his support of both pure and applied mathematics (and the importance of this to the genesis of von Neumann's Electronic Computer Project at IAS), and his love of the outdoors, which led to the acquisition and preservation of what is now the Institute Woods.

The talk concluded with a note of appreciation on behalf of all the families who have enjoyed free run of the institute for seventy-five years. Mathematicians tend to produce their best ideas about the same time that they produce their children. In this, as in so much else, the Institute for Advanced Study (especially through its Housing Project and the Crossroads Nursery School) broke new ground in the cultivation of ideas.

Abstract of Peter Sarnak's talk:

Number Theory, Symmetry and Zeta Functions

An attractive feature of number theory is that the fundamental problems can often be described in elementary terms even though their resolution might require sophisticated mathematical tools. We use two such probing questions of Ramanujan as a theme for a rapid tour of some of the research in number theory carried out at the Institute for Advanced Study. Specifically, we touch on works of the regular faculty members Bombieri, Borel, Deligne, Harish-Chandra, Langlands, Selberg, Siegel, and Weil concerning counting solutions to diophantine equations, zeta functions, and automorphic forms. Their contributions individually and collectively are to a large extent responsible for the advanced state of the subject today.