Armand Borel

1923–2003

BIOGRAPHICAL *Memoirs*

A Biographical Memoir by
Mark Goresky

NATIONAL ACADEMY OF SCIENCES

ARMAND **BOREL**

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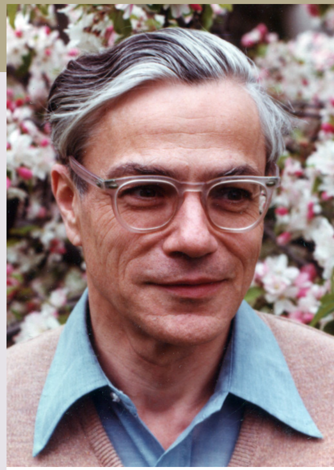

Photo courtesy of the Borel family

By Mark Goresky

Armand Borel was a leading mathematician of the twentieth century. A native of Switzerland, he spent most of his professional life in Princeton, New Jersey, where he passed away after a short illness in 2003.

Although he is primarily known as one of the chief architects of the modern theory of linear algebraic groups and of arithmetic groups, Borel had an extraordinarily wide range of mathematical interests and influence. Perhaps more than any other mathematician in modern times, Borel elucidated and disseminated the work of others. His books, conference proceedings, and journal publications provide the document of record for many important mathematical developments during his lifetime.

Mathematical objects and results bearing Borel's name include Borel subgroups, Borel regulator, Borel construction, Borel equivariant cohomology, Borel-Serre compactification, Bailey-Borel compactification, Borel fixed point theorem, Borel-Moore homology, Borel-Weil theorem, Borel-de Siebenthal theorem, and Borel conjecture.[1]

Borel was awarded the Brouwer Medal (Dutch Mathematical Society), the Steele Prize (American Mathematical Society) and the Balzan Prize (Italian-Swiss International Balzan Foundation). He was a member of the National Academy of Sciences (USA), the American Academy of Arts and Sciences, the American Philosophical Society, the Finnish Academy of Sciences and Letters, the Academia Europa, and the French Academy of Sciences.

1 Borel enjoyed pointing out, with some amusement, that he was not related to the famous Borel of "Borel sets."

Switzerland and France

Armand Borel was born in 1923 in the French-speaking city of La Chaux-de-Fonds in Switzerland. He graduated in 1947 from the Swiss Federal Institute of Technology in Zürich, and obtained a position as an assistant at the same institution, which he held for two years. His prominent teachers Eduard Stiefel and Heinz Hopf ignited his fascination with the interplay between topology and Lie groups.

With a research grant from the French CNRS, Borel moved to Paris for the 1949–50 academic year. It was an exciting moment to be in Paris, then the home of Henri Cartan, Jean Dieudonné, Jean Leray, and an exceptional group of mathematicians under the age of 30 including Jean-Louis Koszul, Roger Godement, Pierre Samuel, Jacques Dixmier, and especially Jean-Pierre Serre. In this stimulating atmosphere Borel's view of the mathematical landscape exploded, and Borel later wrote, [30] "All these people—the elder ones, of course, but also the younger ones—were very broad in their outlook. They knew so much and knew it so well." Borel attended and contributed to the Cartan seminar, and he was inducted into the secret but influential society Nicolas Bourbaki. Borel took careful notes in the course on spectral sequences that his advisor, Jean Leray, gave at the Collège de France. Serre, who credits [60] Borel with teaching him about spectral sequences, began a lifetime of collaboration with Borel and their first joint paper appeared in 1950.

In 1950–52 Borel returned to Switzerland, having obtained a junior position in Algebra at the University of Geneva. During this period he gave a series of lectures in Zürich, essentially repeating Leray's course and distributing his 90 pages of mimeographed notes which were eventually published [26] and are still in print.

In Geneva, Borel completed the writeup of his thesis, in which he applied the new theory of spectral sequences to determine the cohomology (with integer coefficients) of the classifying spaces of Lie groups, one of several results now known as "Borel's theorem." Borel defended the thesis in Paris early in 1952. It became a ninety-page article [4] in the *Annals of Mathematics* and established Borel's style of authoring encyclopedic articles that provide a complete and foundational description of a subject.

Princeton and Chicago

1952 was a pivotal time in Borel's life. Completion of the highly regarded thesis led to a postdoctoral invitation from the Institute for Advanced Study in Princeton. Married only a few months, Borel and his wife, Gaby, arrived in Princeton in the fall of 1952 and spent the next two academic years there. Fritz Hirzebruch happened to be spending exactly the same two years at the IAS and together Borel and Hirzebruch began their enormous joint project [38], [39] that provides a group-theoretic "roots and weights" description of the topology and characteristic classes of homogeneous spaces. See [29] for a description of other exciting developments during these two years.

The resulting calculations using Hirzebruch's Riemann Roch theorem led to a surprising coincidence (between the dimension of an irreducible representation and the dimension of the space of sections of a line bundle on the flag manifold) which Borel described to André Weil [29] when he moved to Chicago for the academic year 1954–55. At that point, says Hirzebruch, "it was only a short step to the Borel-Weil theorem" (see [57], [29]). (Borel and Weil considered compact groups. Claude Chevalley [53] gave the first proof for algebraic groups, cf. [63]).

Borel wrote notes on the result [5], which he intended to publish with Weil. But after receiving a copy of the notes from Borel, Serre reported on the result in Seminaire Bourbaki [61] so Borel dropped the plan to publish the notes. Later, Raoul Bott worked out the higher cohomology generalization of this result [51] and it is now known, with various permutations of the names, as the Bott-Borel-Weil theorem.

In Chicago, Borel absorbed Weil's global view of algebraic geometry and number theory. Admiring Weil's creative genius, Borel once remarked[2] that "Weil always knew what one should do next." Borel also took this opportunity to pursue his love of American jazz, enjoying very late nights at the clubs in Chicago as he had earlier in Zürich and New York City. He added to his growing collection of 78 rpm recordings of jazz performances, eventually amassing an enormous and impressive historical collection.

Borel was prolific over the next few years, but the highlight from this period was his second landmark paper [7], laying the foundations for the modern theory of linear algebraic groups [62]. In this paper, perhaps the first version of a treatise that he would write and re-write throughout his lifetime,[3] Borel systematically applies methods of algebraic geometry to the study of algebraic groups. He makes essential use of the maximal solvable subgroups, now known as Borel subgroups, and gives a beautiful and elegant proof using a new fixed point theorem, now known as the Borel fixed point theorem, that they are all conjugate. After reading an early version of Borel's paper, Chevalley adopted Borel's viewpoint from algebraic geometry ([32] p. 158). He assigned the name

2 in private conversation with R. MacPherson.

3 see [11], [12], [13], [49].

"Borel subgroup" and proved [53], [54], the crucial fact that every Borel subgroup is its own normalizer. Chevalley recounts ([32] p. 158) that his classification of semisimple algebraic groups in terms of the root system then "followed by analytic continuation."

Institute for Advanced Study

In 1957, at the age of 34, Borel received and, after considerable hesitation (see [29]), accepted an offer of a permanent position at the Institute for Advanced Study, joining the Mathematics faculty of Arne Beurling, Deane Montgomery, Kurt Gödel, Marston Morse, Alte Selberg, John von Neumann, and Hassler Whitney, with André Weil joining in 1958. Other professors present at the time included Oswald Veblen (emeritus), Freeman Dyson, Robert Oppenheimer, Chen-Ning Yang, George Kennan, Harold Cherniss, and Homer Thompson. Borel arrived with his family, which now included daughters Anne and Dominique.

Alexander Grothendieck had recently given lectures on his generalized Riemann-Roch theorem. As Serre was visiting the IAS during the fall of 1957, he again joined with Borel to figure out what Grothendieck was saying, and together they created an informal "working" seminar resulting in [44]. As reported in [3], Grothendieck was simultaneously writing his own informal notes [56] on the subject but was dissatisfied with his approach so he did not publish the notes.

Borel also began the first of many high level year-long "learning seminars" dedicated to understanding a topic of current interest and disseminating the results. He and Serre organized a seminar on complex multiplication, resulting in [43]. His continuing interest in Smith theory and his friendship with Deane Montgomery (see [23]) led to the Seminar on Transformation groups [33], that is, the action of a group G on a topological space X. It may not have been clear in 1959 how significant the "Borel construction" $X_G = EG \times_G X$ was to become, but its cohomology $H^*(X_G)$, now referred to as the equivariant cohomology $H^*_G(X)$, is a central object in this seminar. In 1994, Borel remarked to this writer that mathematicians "have simply moved the 'G' from the 'X' to the 'H' and everyone seems to think this is a big deal."

Borel's "learning seminars" at the IAS and later in Bern resulted in additional books on subjects such as Atiyah-Singer [42] (1963-64), discrete subgroups [22] (1968-69), continuous cohomology [47] (19767-77), intersection cohomology [34] (1983) and algebraic D-modules [35] (1984). He was an influential participant in IAS affairs. He believed very much in the unity of mathematics and supported attempts to maintain a wide range and balance of mathematical subjects represented at the IAS despite its limited faculty size (see [3], [29]).

Soon after arriving at the IAS, Borel made acquaintance with John Coleman Moore, an assistant professor at Princeton University at the time. Following up on Borel's earlier note [8] concerning Poincaré duality on generalized manifolds, he and Moore introduced the notion of Borel-Moore homology [40]. Their paper adopts a new point of view using *injective* sheaves and provides a general construction of the *dual of a complex of sheaves*. This paper pre-dates the work of Jean- Louis Verdier [66], who interpreted the Borel-Moore dual of a complex of sheaves A in a more functorial way, as what is now known as the Verdier dual, $D(A) = RHom(A, D_X)$. Here, the *dualizing sheaf* D_X turns out to be the Borel-Moore sheaf of chains on X. (Following suggestions of Grothendieck, Verdier introduced the *derived category* [64], [65], providing a language in which to express the isomorphism $D(D(A)) \cong A$ that Borel and Moore had been unable to prove, or even to formulate.)

1960s and 1970s

In 1961, Borel and his wife, Gaby, visited the TATA Institute in Bombay. Borel lectured there on semisimple groups and Riemannian symmetric spaces, writing notes that were widely distributed and eventually published [10]. Borel's interest in the history of the subject, and his intentions and efforts to write a full account of the subject are described in the introduction.

During this visit, Borel's passion for improvisational music expanded to include Indian music. Over the next forty years, he made many voyages to India, sometimes with the express purpose of attending the annual Madras (now, Chennai) music festival, in which the top south-Indian Carnatic musicians and Bharatnatyam dancers perform in an intense celebration that continues, day and night, for several weeks. When Borel returned, he organized a series of music concerts at the IAS and continued to run these until 1992. The series was adopted by the Artist-In-Residence program in 1994 and was solidified by an endowment from the Edward T. Cone foundation in 2007.

In 1961–62, Harish Chandra, who was recovering from exhaustion due to overwork, visited the IAS. He had recently attended the pioneering lectures in Paris in which André Weil developed the first notions of arithmetic groups. Harish Chandra had since developed reduction theory for adjoint semi-simple groups and thereby proved that arithmetic quotients of such groups have finite volume, a result that in his opinion "would have made Poincaré happy" [58]. Borel and Harish Chandra began to discuss reduction theory and in Borel's signature style, two encyclopedic papers [20], [21] emerged, laying the foundations of the theory of arithmetic groups, the only jointly authored mathematical publications of Harish Chandra. Borel continued to pursue the foundations of the theory (see [60]). Notes from his 1964 course at the Institut Henri Poincaré became his book [14], which remains the standard reference on arithmetic groups.

When Jacques Tits visited the IAS in 1962–63, he and Borel realized they had many overlapping results, sometimes from different perspectives, and so they decided to put their ideas together, resulting in their exhaustive treatise [46], commonly referred to as "Borel-Tits," on reductive groups over non algebraically closed fields.

Throughout the 1960s, Borel wrote many further papers on the foundations of the theory of algebraic and arithmetic groups and by the end of the decade he had published the next version [12] of his reference work on algebraic groups.

The quotient $X = \Gamma\backslash D$ of a Hermitian symmetric space D by an arithmetic group Γ has an obvious complex analytic structure. A fundamental construction of Borel and W. Baily [1], [2] from this period provides an embedding of X into complex projective space, simultaneously endowing X with an *algebraic* structure, and a natural algebraic compactification, now called the *Baily-Borel* compactification. Goro Shimura, Pierre Deligne, James Milne, and others constructed "canonical" models over number fields and the resulting *Shimura varieties* have become central objects of study in the Langlands program. Borel later showed that a holomorphic map from an algebraic variety to X is automatically algebraic.

Smooth (manifold with corners) compactifications of locally symmetric spaces are constructed in great generality in the classic paper of Borel and Serre [45]. Consequently, such spaces do not exhibit infinite topological complexity, and in fact, most approaches to the topology of locally symmetric spaces begin with the Borel-Serre compactification. Borel continued to explore the cohomology of locally symmetric spaces in [45], in which the Borel stability theorem is proven and the Borel regulator in K-theory is constructed. Alexander Beilinson later generalized this construction and Borel's theorem in [27], identifying the regulator with the value of a zeta function at integer points, may be viewed as a proof of a special case of Beilinson's conjectures, cf. [55].

Borel organized and co-organized conferences almost every year, but two of these stand out as having been highly influential. In 1965, Borel, together with George Mostow, organized the AMS Summer Math Institute on arithmetic aspects of algebraic groups in Boulder, Colorado. The resulting book [41] is filled with articles that are now classics. Tonny Springer notes [63] that "one is struck by the taste and foresight shown in the choice of the subjects."

Borel, together with Bill Casselman, organized the 1977 Corvallis conference on Automorphic Forms, Representations, and L-functions [37]. This meeting had an enormous effect on generating interest in the field of automorphic forms and on the dissemination of Langlands' ideas. Survey papers and background articles constitute over half the articles in the two-volume conference proceedings, which have become essential reading for devotees of automorphic forms.

Retirement

As Borel's mandatory retirement date drew near, he considered returning permanently to Switzerland and consequently accepted a position at the ETH Zürich, 1983–1986. During this period, he and André Haefliger organized a series of seminars jointly with the Department of Mathematics at the University of Geneva. The "Borel Seminar," which continues to thrive, would meet in the "University Room," a conference room in the Bern train station, thereby accessible by morning train from Geneva and Zürich. It produced volumes on Intersection Cohomology [34] and Algebraic D-modules [35], but in the end Borel decided to remain at the IAS, where he was guaranteed continued use of his office and secretary.

Borel, together with his host Ngaiming Mok, created a Program on Lie Groups at Hong Kong University, which ran for four months in each of the years 1999, 2000, and 2001 (see [59]). It was a carefully orchestrated program with lectures and courses by Borel and others, starting with basic material in the first year and ending with special topics in the third year.

In his later years Borel explained to his friends that he had thought carefully about activities in which an aging mathematician could continue to contribute to the profession. Although he continued with his research, during this period he also wrote a collection of historical and biographical articles. "Mathematics, Art and Science" [15], [16] is a fascinating and eloquent portrayal of the nature of mathematics and its place in the world (cf. [3]). His book [32] of historical essays is an ambitious work that covers developments from Lie to Chevalley.

Borel's biographical/historical notes on Deane Montgomery [23], Jean Leray [36], [26], and André Weil [25] concern the history of topology; his notes on Hermann Weyl [19], Claude Chevalley [18], [32], and Ellis Kolchin [18], [32] involve the history of Lie groups and algebraic groups, and his carefully researched article [31] on Henri Poincaré addresses the origins of special relativity. His historical articles on Bourbaki [30] and on the Institute for Advanced Study [29] and his tributes to Harish Chandra [17} and André Weil [24] contain many fascinating and amusing personal recollections.

Borel the man

Borel viewed mathematics as the greatest of human achievements, a gigantic structure forged over a thousand years by many of the most talented and dedicated people on the planet. He was a rationalist nonpareil, having once remarked, perhaps facetiously, upon entering the magnificent Gothic cathedral at Chartres, on the awesome power of superstition.

He had an uncompromising view of his responsibility to uphold integrity and he expected the same of those around him. Borel could be intimidating and unapproachable, especially on a first meeting. At other times he would let down his guard and his ironic sense of humor would emerge. As Chandrasekharan recalls [52], "He detested the display of self-importance or officiousness in any form…He had a highly developed sense of the absurd, which moved him to outright laughter when faced with people who spoke or wrote about things they did not know." But despite his sometimes austere professional demeanor, Borel was a warm and supportive mentor who cared deeply for the disadvantaged.

Many mathematicians will recognize as typical, Nolan Wallach's experience during a 1969 party when he collected his nerve, walked up to Borel and said "Professor Borel, I would like to introduce myself. My name is Nolan Wallach." "Wallach...Wallach..." Borel hesitated, "Ah! Vallack!" now recognizing the name and prounouncing it with a 'V' and a German 'ch', "Your Theorem 8.3 is false." Wallach went over to his wife Barbara, saying he wasn't feeling well, perhaps they should go home now [67]. But Wallach goes on to say that Borel later contacted him, proposing that they run a seminar together, the results of which eventually became their classic book [47].

Borel and Enrico Bombieri, for example, regularly visited John Nash when he was recovering at home from mental illness.

Borel exercised vigorously, increasing his regimen as he became older. He loved the mountains and organized hikes at many conferences. He was a competitive hiker and even in his seventies would forge ahead of the crowd until, reaching the summit alone, he would turn and look back with a smirk, one hand on his hip, as if to say "Well, come on, I've been waiting for you."

The written word

In Borel's view, mathematical enterprise is built upon the published record. He served as editor for the *Annals of Mathematics* (1962–79) and *Inventiones Math* (1979–93). He required that every conference or summer school should publish a proceedings containing background and basic material. He wrote many definitive articles on topics of current interest and he once scoffed at magazine and newspaper writers who produce text that is "designed to be read once, on that day or week, and then forgotten." He was an ardent fan of Sir Arthur Conan Doyle, and he sometimes referred to the much loved proprieter of the IAS guest house as "the unfortunately named Mrs. Moriarty." He chose books for his daughters to read, and he once sent a personal letter to J.K. Rowling, author of the *Harry Potter* books, congratulating her on "significantly raising the literacy rate among youngsters" [48].

In some ways, the epitome of his dedication to the printed word is illustrated by his participation in the anonymous works of N. Bourbaki. The unusually accessible and influential Groupes et Algèbres de Lie iv, v, vi [49] were largely written by Borel, together with Serre and Tits. Borel recalled the events [30] as follows:

> *What remains most vividly in my mind is the unselfish collaboration over many years of mathematicians with diverse personalities toward a common goal, a truly unique experience, maybe a unique occurrence in the history of mathematics. The underlying commitment and obligations were assumed as a matter of course, not even talked about, a fact which seems to me more and more astonishing, almost unreal, as these events recede into the past.*

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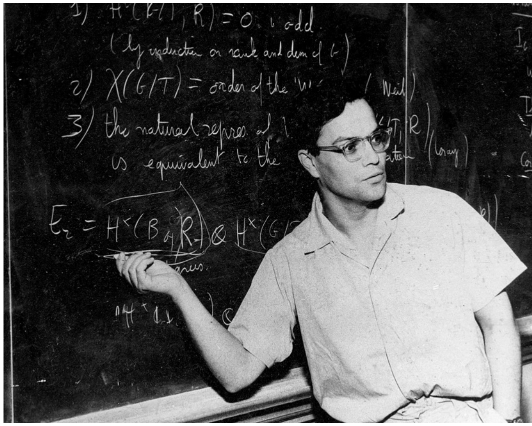
Borel on Mount Speer, 1983.
(Photograph by Bill Casselman.)



Borel at the IAS.
(Photo courtesy of the Borel family.)

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